



**GT and SDR Studies for Stable Nuclei  
and Perspective for Unstable Nuclei**

**Tomotsugu Wakasa**

*Department of Physics, Kyushu University*



# Contents

## COURSE

- **GT study for  $^{90}\text{Zr}$** 
  - Unified understanding of spin-isospin responses
    - $\pi + \rho + g'$  model
    - Two free parameters:  $g'_{\text{NN}}$  and  $g'_{\text{N}\Delta}$
  - Peak position  $\rightarrow g'_{\text{NN}}$  (short-range repulsion in NN)
  - Strength (quenching)  $\rightarrow g'_{\text{N}\Delta}$  (short-range repulsion in  $\text{N}\Delta$ )
  - **No free parameter in  $\pi + \rho + g'$  model**
- **GT/SD study for  $^{208}\text{Pb}$** 
  - Comparison with predictions in  $\pi + \rho + g'$  model
  - **Quantitative reproduction by  $\pi + \rho + g'$  model**
- **Perspective for unstable nuclei**
  - **Can we deduce GT/SD strengths ?**
    - Without performing multipole decomposition
    - Without normalizing to beta-decay strengths
  - **Predictions for Zr and O isotopes in  $\pi + \rho + g'$  model**

# Sum Rule and Giant Resonances

- **Giant resonances**

- Collective motion
- Common to many-body quantum system

- **Unique feature of nucleus**

- Nucleus consists of nucleons with
  - Spin 1/2
  - Isospin 1/2
- $2 \times 2 = 4$  degrees of freedom

- **Resonance strength depends on**

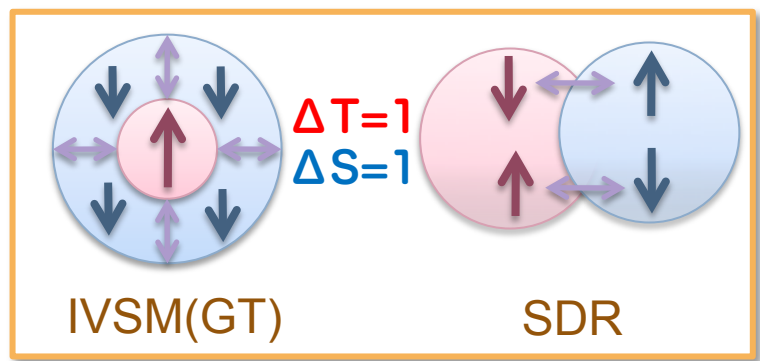
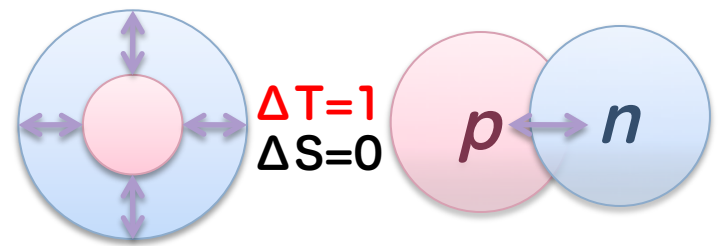
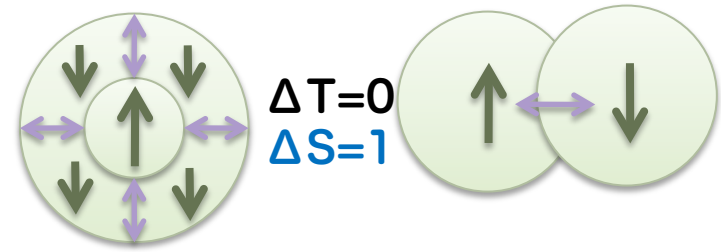
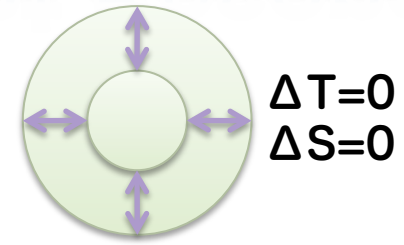
- Number of participating nucleons
- Size of the system
- **Sum-rule depending on g.s. properties**

- **Compare GR strength to sum-rule**

- Residual interaction (distribution)
- Quark degrees of freedom (quenching)

L=0

L=1





# Spin-Isospin Modes and Sum Rule

- Spin-isospin transition operators

Spin-scalar  $O_J^{\tau\pm} = \sum_{i=1}^A r_i^L Y_L(\hat{r}_i) t_i^{\pm}$

Spin-vector  $O_J^{\sigma\tau\pm} = \sum_{i=1}^A r_i^L [Y_L(\hat{r}_i) \otimes \vec{\sigma}_i]_J t_i^{\pm}$

- Model-independent sum-rule

$$S_J^- - S_J^+ = \frac{(2J+1)}{4\pi} \left( N \langle r_n^{2J} \rangle - Z \langle r_p^{2J} \rangle \right) \quad (\sim S_J^- \text{ for } N \gg Z)$$

*n-radius*                      *p-radius*

- Fermi and GT sum rule

Fermi

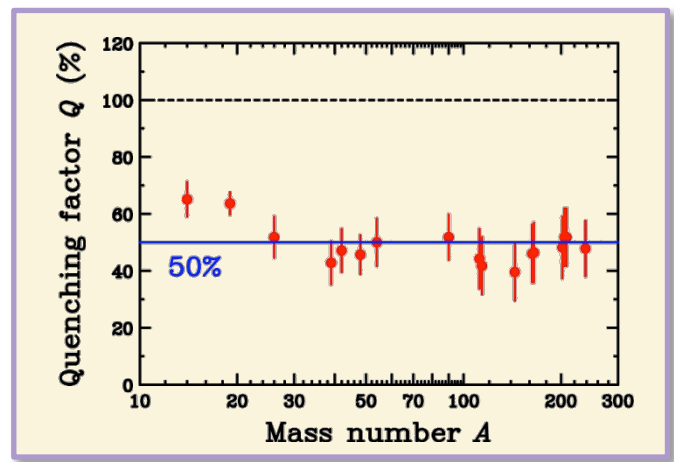
$$S^-(F) - S^+(F) = N - Z$$

Gamow-Teller

$$S^-(GT) - S^+(GT) = 3(N - Z)$$

50% quenching of GTGR

2p2h (Config. Mix.)  
Quark ( $\Delta$ )





# $\pi + \rho + g'$ model and $\Delta$ Effects

- Effective Interaction

$$V_{\text{eff}} = V_L + V_T$$

Longitudinal ( $\pi$ )

Transverse ( $\rho$ )

- $\pi + \rho + g'$  model

- Spin-longitudinal  $V_L = V_L^\pi + V_L^{\text{LM}}$
- Spin-transverse  $V_T = V_T^\rho + V_T^{\text{LM}}$

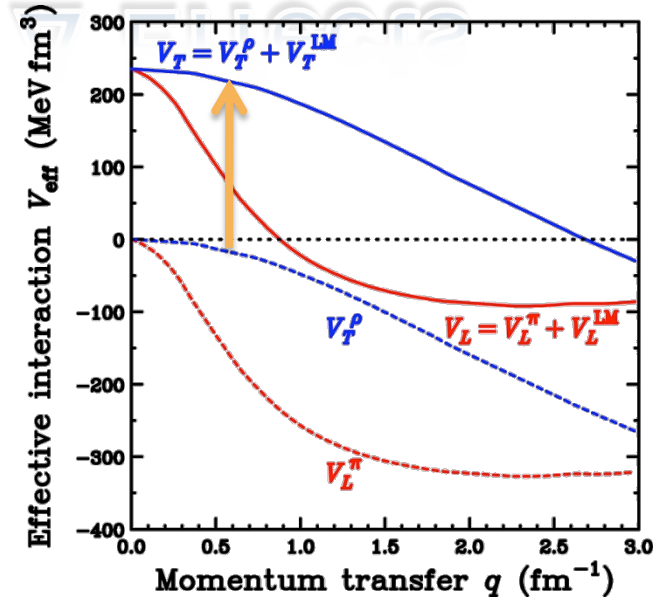
- NN(p-h) effective Interaction

$$V_L(q, \omega) = \frac{f_{\pi NN}^2}{m_\pi^2} \left( \underbrace{\frac{q^2}{\omega^2 - q^2 - m_\pi^2} \Gamma_{\pi NN}^2}_{\pi\text{-exchange}} + \underbrace{g'_{NN}}_{\text{Short-range repulsion}} \right) (\tau_1 \cdot \tau_2)(\sigma_1 \cdot \hat{q})(\sigma_2 \cdot \hat{q})$$

$$V_T(q, \omega) = \frac{f_{\pi NN}^2}{m_\pi^2} \left( \underbrace{C_\rho \frac{q^2}{\omega^2 - q^2 - m_\rho^2} \Gamma_{\rho NN}^2}_{\rho\text{-exchange}} + \underbrace{g'_{NN}}_{\text{Short-range repulsion}} \right) (\tau_1 \cdot \tau_2)(\sigma_1 \times \hat{q})(\sigma_2 \times \hat{q})$$

- Extension to N+ $\Delta$  system for LM interaction

$$V_{N\Delta}^{\text{LM}} = \frac{f_{\pi NN} f_{\pi N\Delta}}{m_\pi^2} g'_{N\Delta} \left\{ \begin{array}{l} g'_{NN} : \text{Strength "distribution"} \\ g'_{N\Delta} : \text{Strength "quenching"} \end{array} \right.$$



# GT Strength and LM Parameters

K.Yako et al., PLB 615(2005)193. T.W. et al., PRC 72(2005)067303.

- $g'$  dependence on GTGR
  - RPA(1p1h) by Ichimura group
  - GTGR peak position

- Strongly depends on  $g'_{NN}$

$$g'_{NN} = 0.6 \pm 0.1$$

- Weak  $g'_{N\Delta}$  dependence

- $g'_{N\Delta}$  dep. on GT quenching Q

- $Q = 0.86 \pm 0.07$

- 2p2h effects are dominant

- Q evaluated in RPA

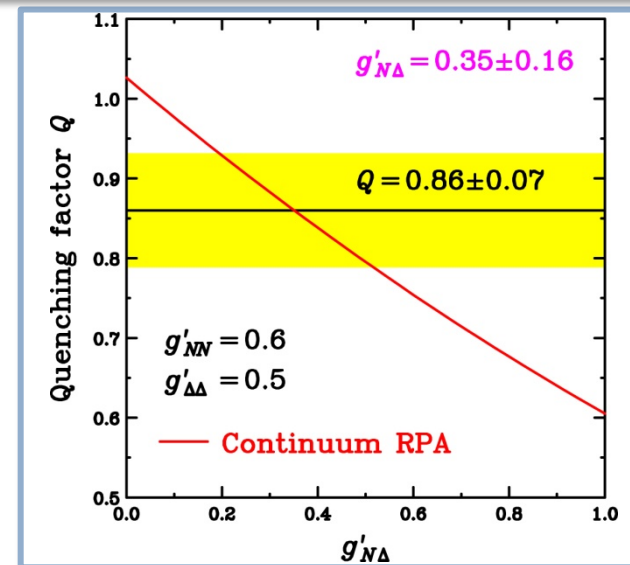
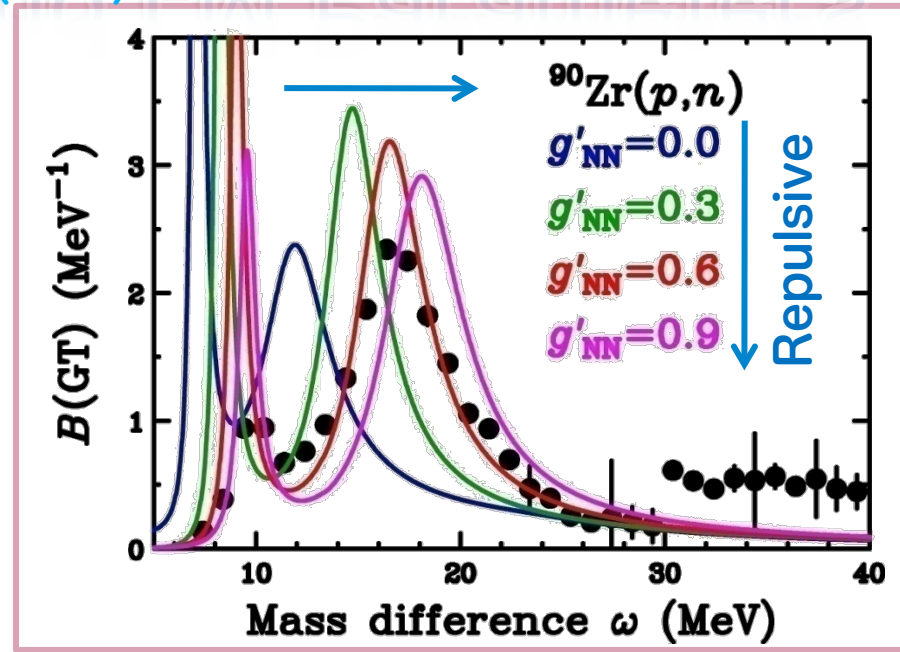
- Strongly depends on  $g'_{N\Delta}$

$$g'_{N\Delta} = 0.35 \pm 0.16$$

- How about other modes (resonances)

- Quenching (?)

- Distribution (Information on residual Int.)



# Correlation and $\Delta$ Effects on SDR

- SDR in  $0^{\text{th}}$

- $E_x(2^-) < E_x(1^-) < E_x(0^-)$
- Reflecting shell-structure
  - $B.E.(j_>) > B.E.(j_<)$

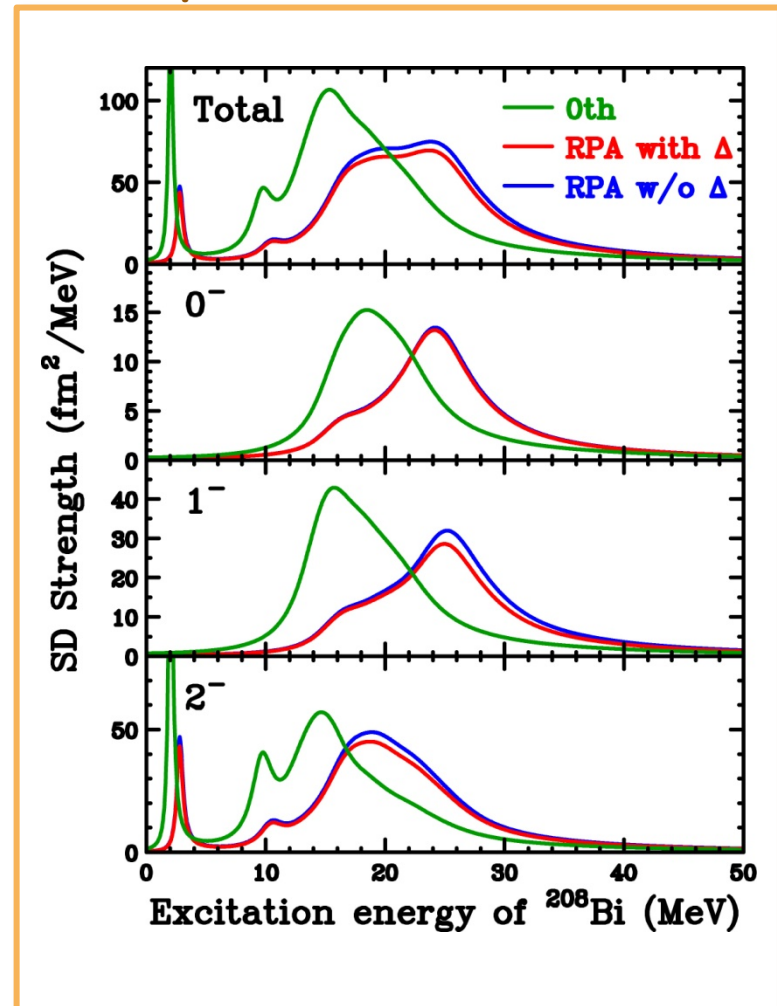
- SDR in RPA

- NO free parameters
  - Same  $g'$  determined by GT
- $E_x(2^-) < E_x(1^-) < E_x(0^-)$ 
  - Same as  $0^{\text{th}}$
- Move strengths to higher  $E_x$ 
  - Repulsive p-h int.

- $\Delta$  effects

- Very small in SDR

$^{208}\text{Pb}(p,n)$



# Previous Studies of SDR

- Multipole decomposition of  $^{90}\text{Zr}(^3\text{He},t)$  at 900 MeV

$$\begin{aligned} \text{Exp.} \quad \sigma(\omega, \theta) &= \text{Fit} \quad \text{DW calc.} \\ &+ a_0(\omega) \sigma_{L=0}(\theta) \\ &+ a_1(\omega) \sigma_{L=1}(\theta) \\ &+ a_2(\omega) \sigma_{L=2}(\theta) \end{aligned}$$

- $\theta = 0.25^\circ \sim 4.25^\circ$

- SDR cross section

- Quenching ( $\sim 30\%$ ) from RPA(1p1h)

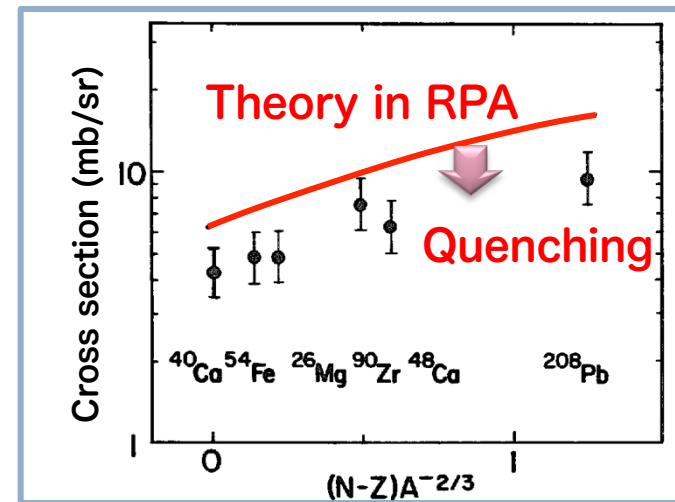
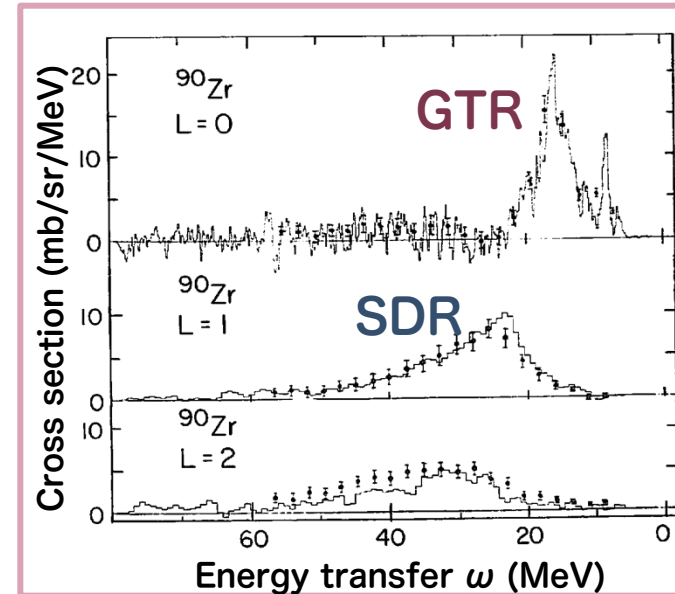
- 2p2h (Configuration mixing)
- Other mechanism

- Spin-parities could NOT be separated

- Similar angular distributions of 0<sup>-</sup>, 1<sup>-</sup>, 2<sup>-</sup>

- NOT conclusive for quenching

- Rough estimation for distortion effects (NOT in full DWIA)
- RPA in g' only (w/o  $\pi/\rho$ -exchange)
- Angular distributions in ( $^3\text{He},t$ ) are steep

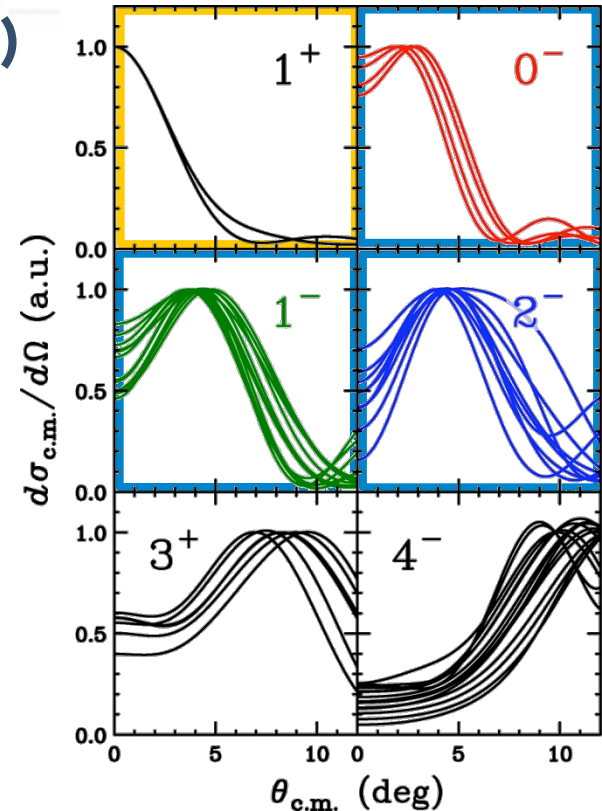




# MD Analysis for SD

## – Difficulties and Solutions –

- **Multipole Decomposition Analysis (MDA)**
  - L-dependence of angular distributions
    - Insensitive to  $J^\pi$
- **MDA for GT**
  - $0^+$  and  $1^+$  for  $L=0$ 
    - $0^+$  strength  $\rightarrow$  IAS (Easily removed)
- **MDA for SD**
  - $0^-, 1^-, 2^-$  for  $L=1$ 
    - Separation is difficult with  $\sigma$
  - GDR and SDR for  $1^-$  could not be separated
- **Polarization transfer  $D_{ij}$  for SD**



	$D_{NN}(4.0^\circ)$	$D_{LL}(4.0^\circ)$
$0^-$ (SDR)	-1.00	-1.00
$2^-$ (SDR)	-0.17	-0.41
$1^-$ (SDR)	+0.19	-0.16
$1^-$ (GDR)	+0.96	+0.95

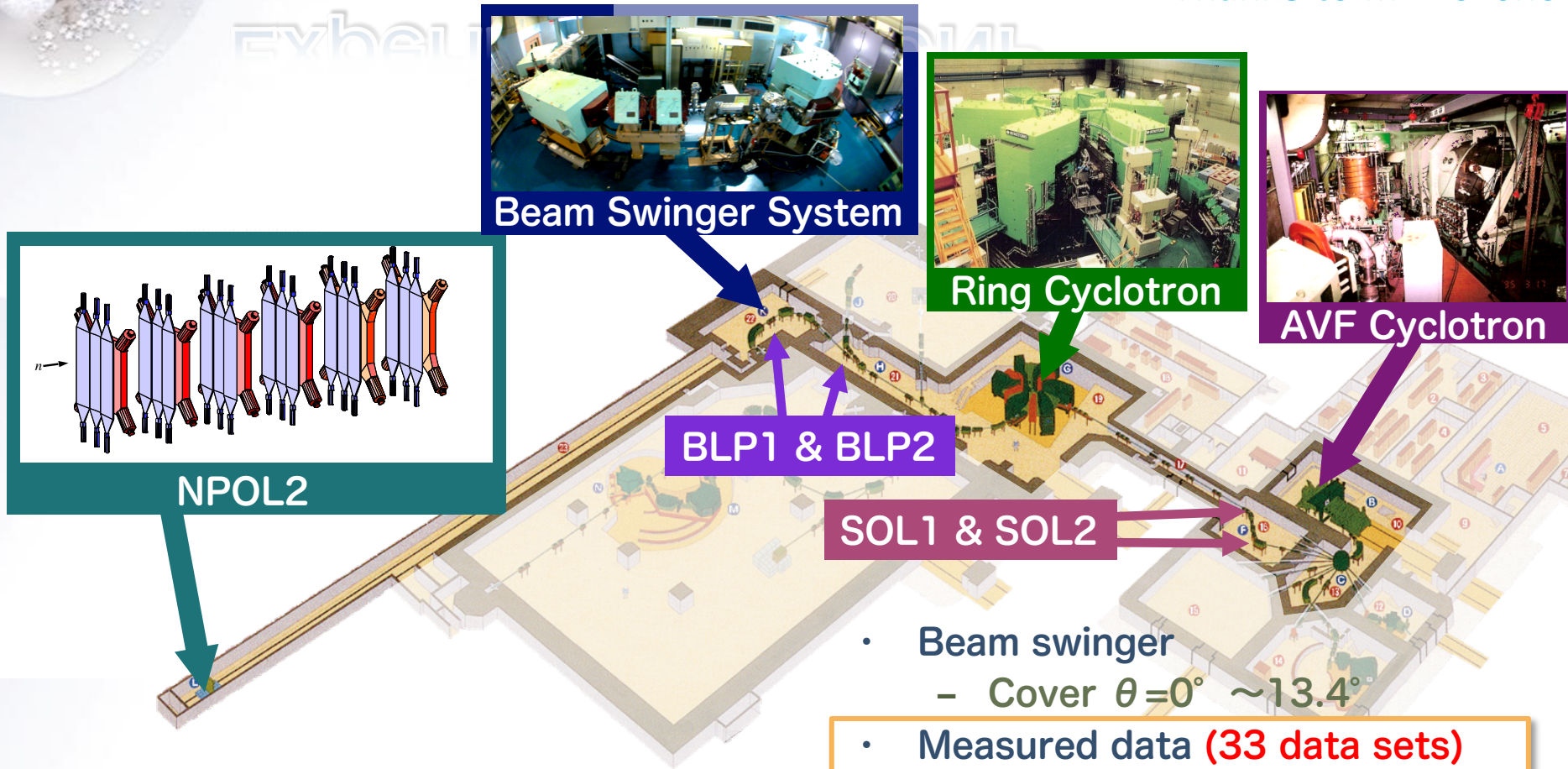


### MDA with $D_{ij}$

- ✓ Separate  $0^-, 1^-, 2^-$
- ✓ Separate GDR and SDR

# Experiment at RCNP

Thanks to M. Dozono



- 295 MeV Polarized protons
  - Predominantly excite GT and SDR
- Beam polarization
  - Control with 2-sets of solenoids
  - Measure with 2-sets of BLP by p-p

- Beam swinger
  - Cover  $\theta = 0^\circ \sim 13.4^\circ$
- Measured data (33 data sets)
  - $\sigma$  : 13 angles
  - $A_y$  : 12 angles
  - $D_{NN}$  : 4 angles(0,4,7,10deg)
  - $D_{LL}$  : 1 angle(0deg)
  - P : 3 angles(4,7,10deg)

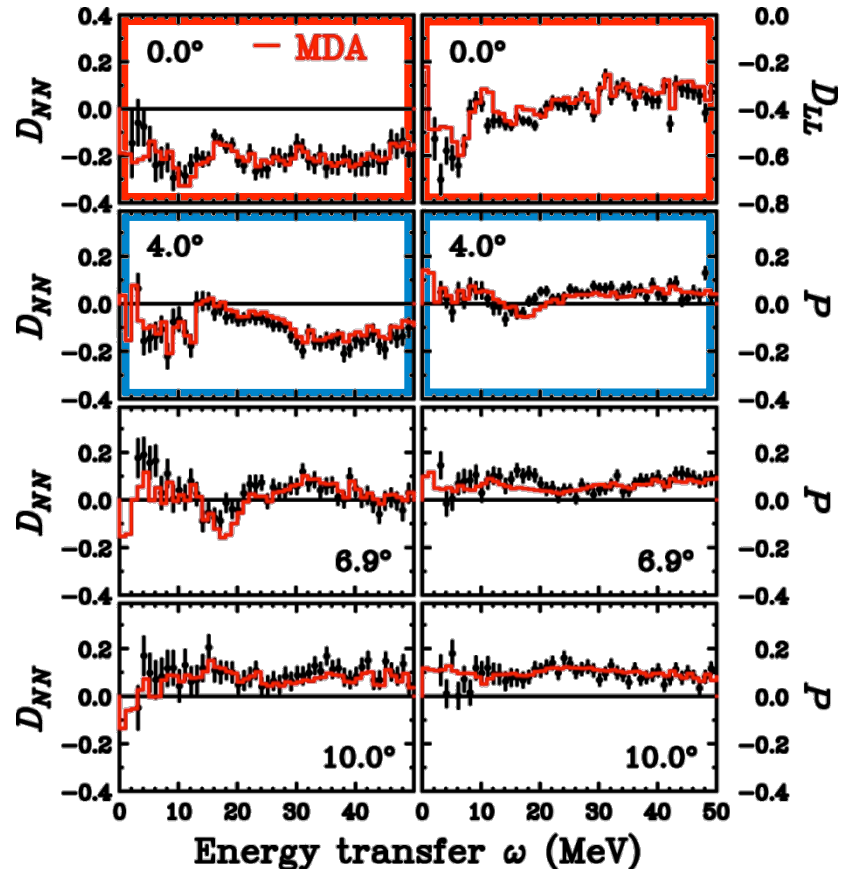
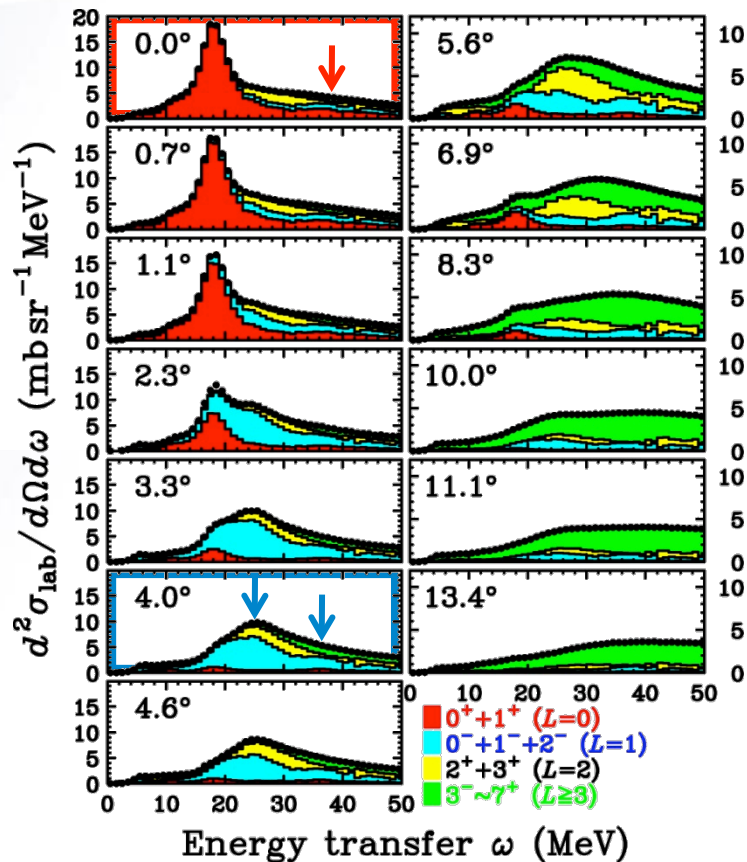


# Results of MDA

~First MDA with Polarization Data~

- MDA (up to 7+(L=6))

- Both cross section and polarization data are well reproduced
- At 0° : Significant L=0 (GT+IVSM) up to 50 MeV
- At 4° : Significant L=1 (SDR) around 20 and 35 MeV (2p2h?)



# Gamow-Teller Strength $B(GT)$

- MDA could not separate GT from IVSM

- Assumption

- Proportionality between GT/IVSM strength and cross section

$$\sigma(\text{GT} + \text{IVSM}) = \hat{\sigma}_{\text{GT}} [B(\text{GT}) + B(\text{IVSM})] F(q, \omega)$$

MDA

GT unit c.s.

- Weak interference between GT and IVSM
    - Similar quenching effects on IVSM

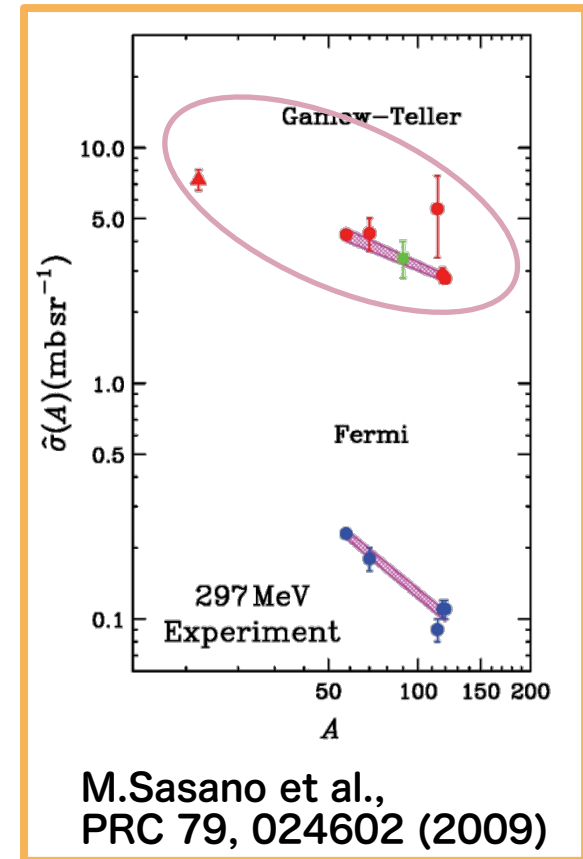
$$\Rightarrow \frac{\sigma^{\text{Exp}}(\text{GT} + \text{IVSM})}{\sigma^{\text{Theor}}(\text{GT} + \text{IVSM})} = \frac{B^{\text{Exp}}(\text{GT})}{B^{\text{Theor}}(\text{GT})}$$

- Reliability for theoretical calculations

$$\hat{\sigma}_{\text{GT}}^{\text{Exp}} = 1.88 \pm 0.17 \text{ mb/sr}$$

$$\hat{\sigma}_{\text{GT}}^{\text{Theor}} = 1.94 \pm 0.16 \text{ mb/sr}$$

Theoretical calculations are reliable  
 → Systematic uncertainty of  $B(GT) \sim 10\%$



# GT Strength $B(\text{GT})$

~Comparison with 2p2h calc.~

- **Experimental  $B(\text{GT})$**

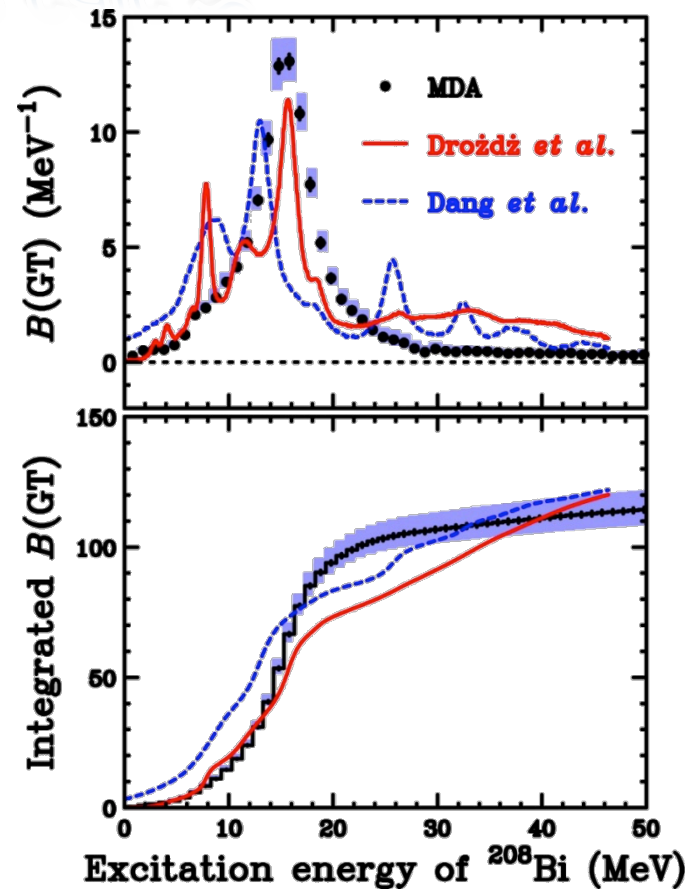
- **Strength up to 50 MeV**
- Not significant compared with  $^{90}\text{Nb}$

$$S^-(\text{GT}) = 115 \pm 1(\text{stat}) \pm 7(\text{MDA})$$
$$= \underline{87 \pm 5\%}$$

- Configuration mixing is dominant
  - Quark ( $\Delta$ ) effect is small
- Consistent with  $Q=0.86$  for  $^{90}\text{Nb}$ 
  - $S^+(\text{GT})$  is expected to be small

- **Theoretical calc. with 2p2h**

- **$S(\text{GT})$  is consistent**
- **Different  $B(\text{GT})$  distributions**
  - Exp: Concentrate in GR region
  - Theory: Significant spread



S.Drozd et al., PLB 189, 271(1987)  
N.D.Dang et al.,PRL 79,1638(1997)

Further studies are required for conclusions

# SD Unit Cross Section

*C. Gaarde et al., NPA 369, 258 (1981)*  
*K. Yako et al., PRC 74, 051303(R)(2006)*

## • Relation between SD cross section and SD strength $B(\text{SD})$

- Proportionality ansatz

$$\frac{\sigma_{\text{SD}, J\pi}(4.0^\circ)}{\text{MDA}} = \frac{\hat{\sigma}_{\text{SD}, J\pi}}{\text{Unit c.s. Strength}} B(\text{SD})$$

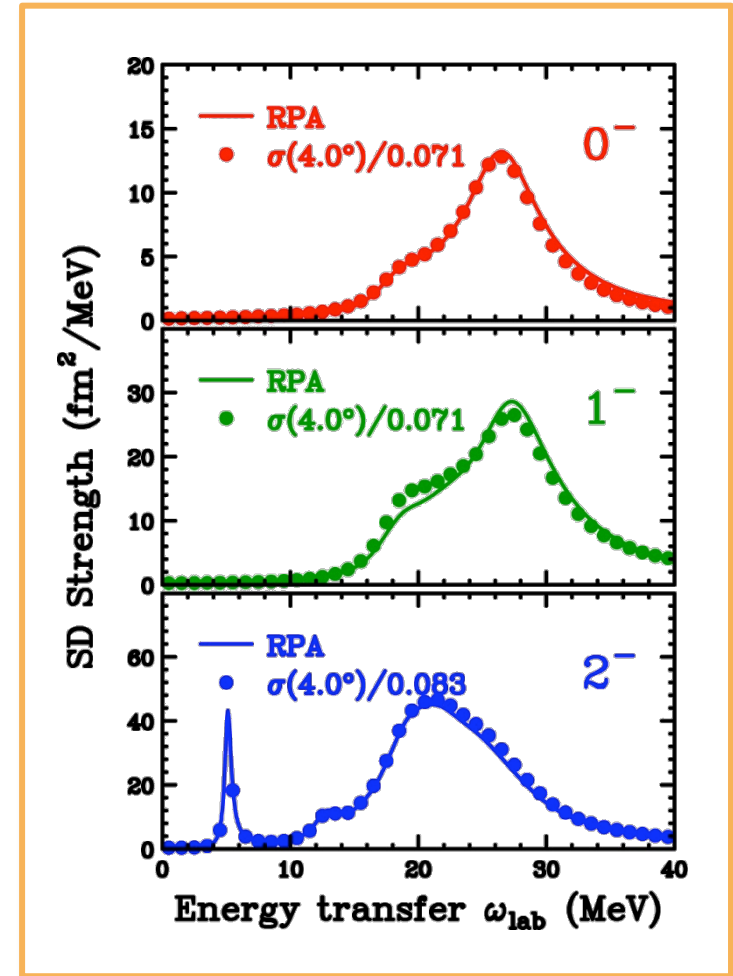
- **Proportionality** :  $\sim 10\%$
- Differences from constant unit c.s.
  - q-dep. (q is a function of  $\omega$ )
  - Structure (radial W.F.)
  - Tensor int. (structure dep.)

$$\hat{\sigma}_{\text{SD}, J\pi} \rightarrow \hat{\sigma}_{\text{SD}, J\pi}(\omega)$$

based on DWIA+RPA calc.

## • Unit c.s. depends on DWIA inputs

- Optical potential, etc.
- **Uncertainty** :  $\sim 10\%$





# SD Strength Distributions

~Comparison with RPA~

- **Experimental B(SD)**

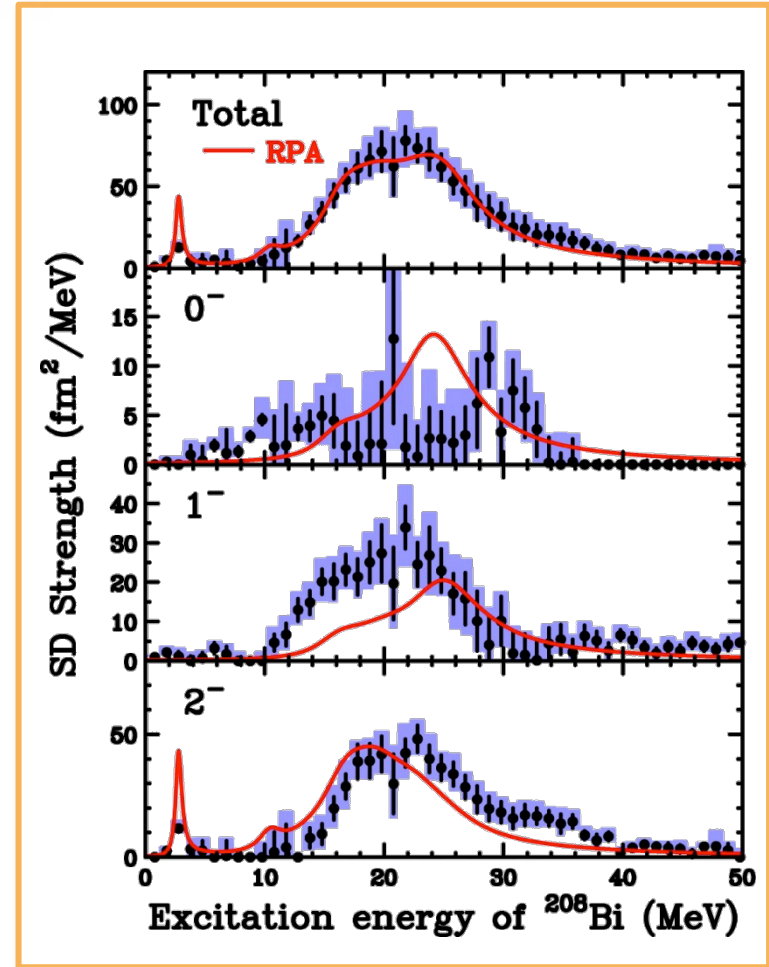
- Asymmetric single peak for 1- & 2-
  - Tail to higher  $E_x$  up to 40 MeV
- Fragmented 0- strength

- **Theoretical calc. in RPA**

- Phenomenological spreading width
  - Effective inclusion of 2p2h (in part)
- Strength distribution
  - Total strength is consistent
  - 0- strength is severely fragmented
- Sequence of SDR peak
  - Exp:  $E_x(2^-) \sim E_x(1^-) \sim E_x(0^-)$
  - Theory:  $E_x(2^-) < E_x(1^-) < E_x(0^-)$

- **Comment on tensor correlations**

- Repulsive effect on 1- : N.G. (attractive effect ? )



# Integrated SD Strength

~Comparison with RPA~

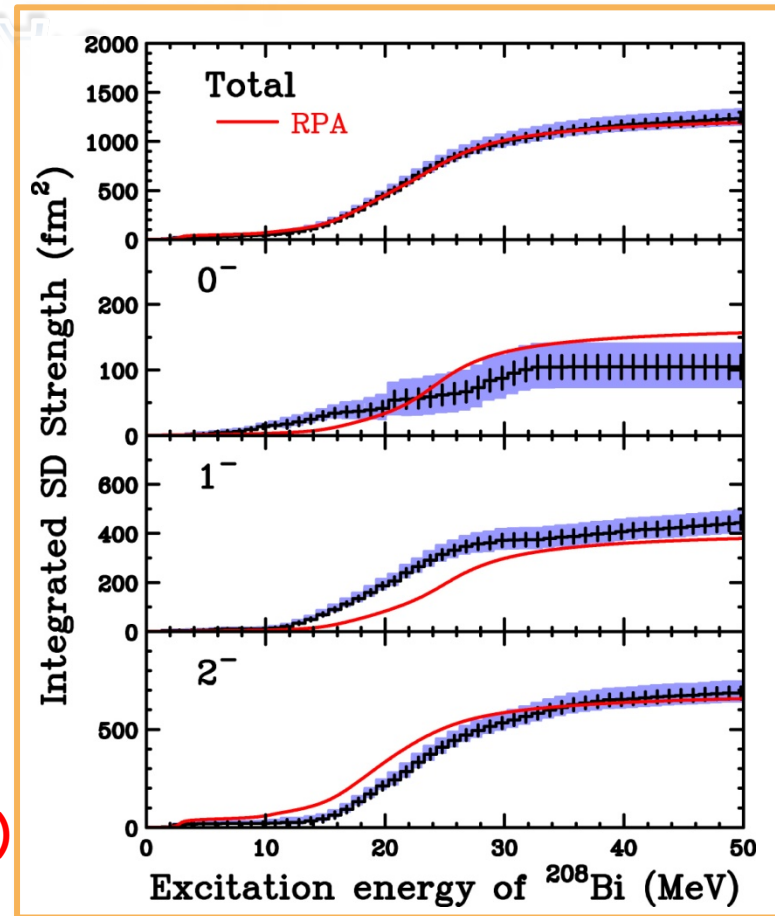
- **Experimental B(SD) from MDA**

- **Uncertainties**

- — : Statistical uncertainty
- : MDA uncertainty
- ~10% systematic uncertainty from  $\sigma_{SD}$

- **Comparison with RPA**

- $0^-$  : Slightly small (NOT significant)
- $1^-$  : Consistent (Softened)
- $2^-$  : Consistent (Hardened)
- **Total : Consistent (Similar distribution)**

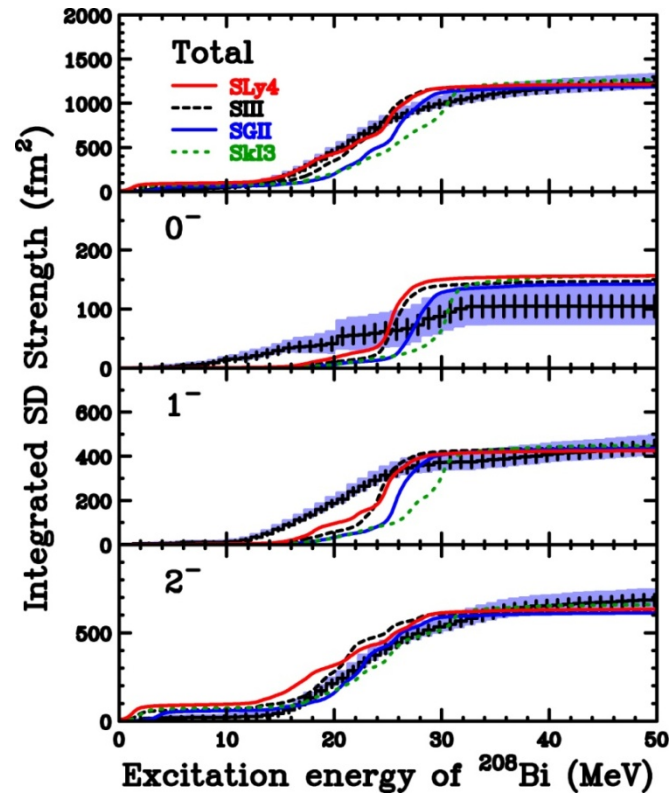
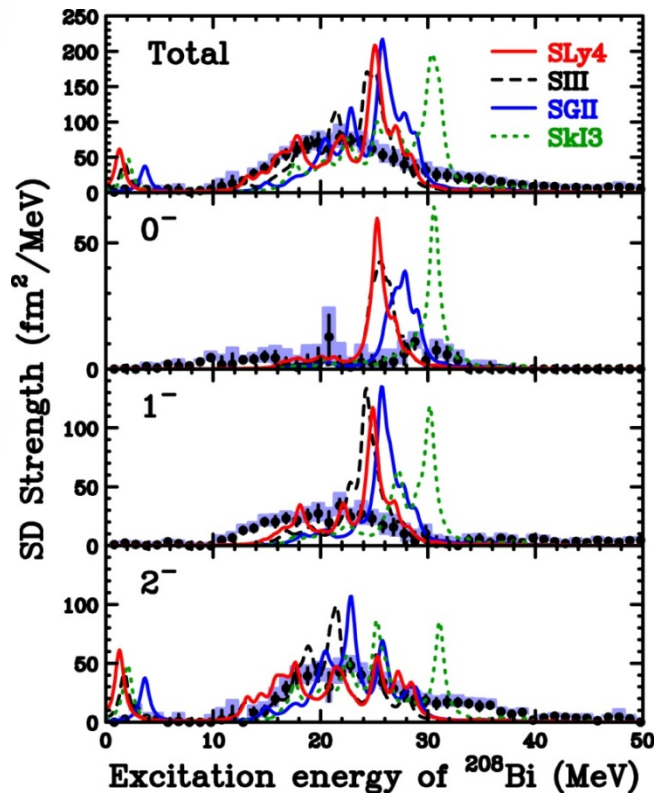


**SD strengths are NOT quenched**

- SD distribution of each  $J^\pi$  is different from theoretical predictions
- $J^\pi$  decomposition in MDA with polarization data is successful

# Comparison with HF+RPA

- HF+RPA calculations by Sagawa-san's group
  - Skyrme interaction: SLy4, SIII,SGII,SkI3
    - 0- and 1- strengths are smeared/fragmented compared with calc.
    - 1- strength is significantly softened
    - Integrated strengths (each  $J^\pi$ , Total) are consistent





# Summary for GT/SD strengths of Stable Nuclei

- **Gamow-Teller strength**
  - Peak position  $\rightarrow g'_{NN} \sim 0.60$
  - Strength (quenching)  $\rightarrow g'_{N\Delta} \sim 0.35$
  - Pionic correlations at large  $q$  are also reproduced with these  $g'$
- **Spin-Dipole strengths**
  - Total SD strengths are well reproduced with present  $g'$  values
    - Spin composition (0-, 1-, 2-) seems to be different
- **Extension to GT/SD strengths of unstable nuclei**
  - MDA would be unsuccessful due to poor statistics
    - **GT/SD strength without MDA ?**
      - Physical background in cross section
    - Connection from cross section to GT/SD strength
      - **Can DWIA reproduce absolute values of cross sections?**
        - YES  $\rightarrow$  Strengths could be deduced from cross sections
- **Predictions for Zr and O isotopes**

# Comparison with MDA and DWIA for $^{90}\text{Zr}(p,n)$

## • MD vs. DWIA

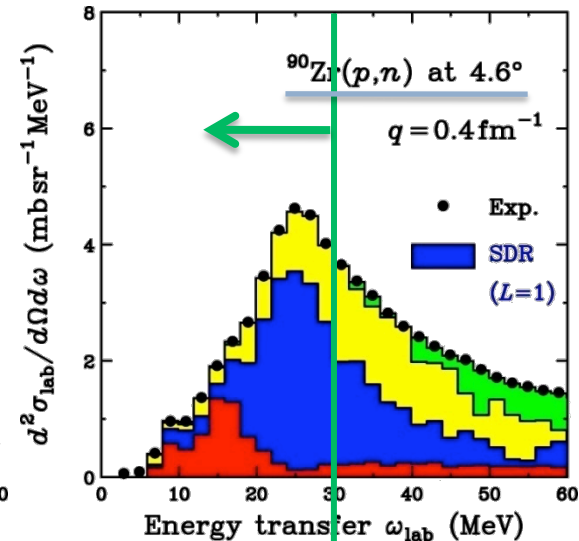
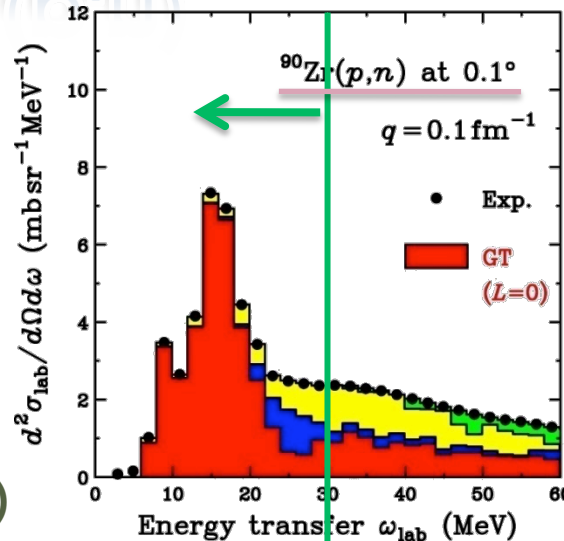
- Consistent
- **B(GT) and B(SD) can be deduced**
- **Negligible 2p-2h effects at  $\leq 30$  MeV**

## • GT

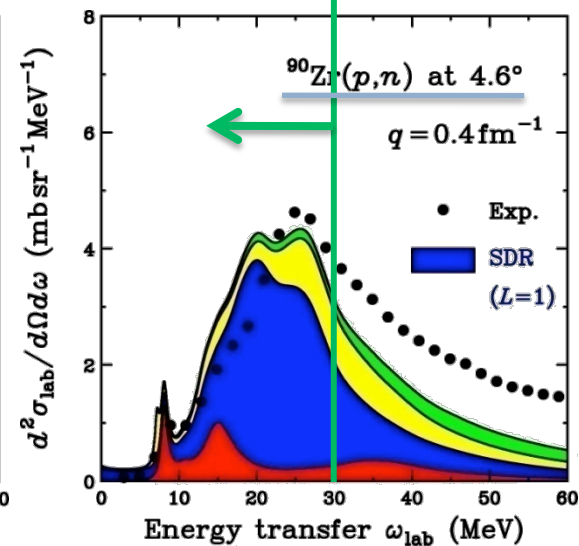
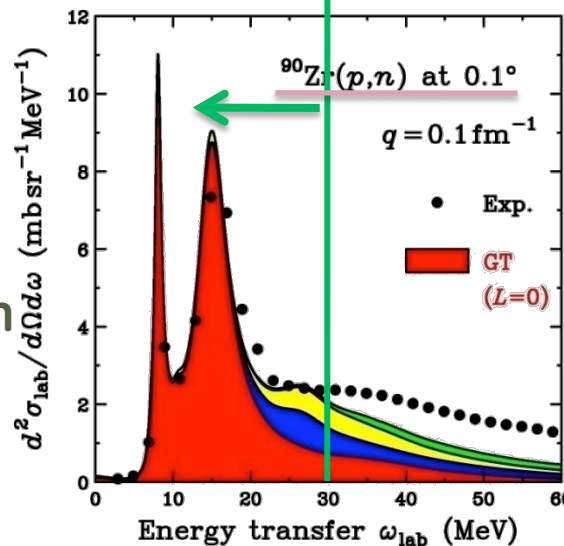
- Negligible B.G. ( $L \geq 1$ ) in GTGR
- **B(GT) can be deduced w/o MDA (Not S(GT))**

## • SD

- Small (10-20%) BG in SDR
- **B(SD) would be deduced w/o MDA with  $\sim 10\%$  error**



MDA



DWIA

# Comparison with MDA and DWIA for $^{208}\text{Pb}(p,n)$

## • MD vs. DWIA

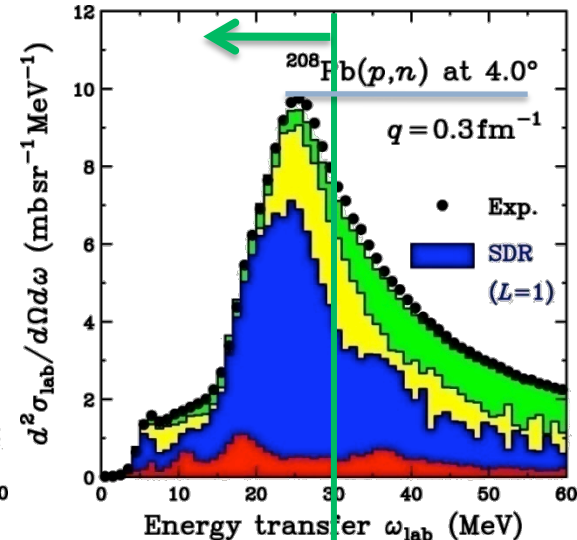
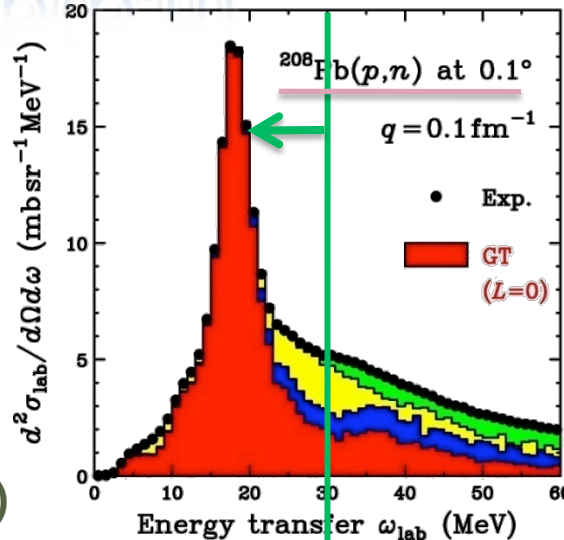
- Consistent
- **B(GT) and B(SD) can be deduced**
- **Negligible 2p-2h effects at  $\leq 30$  MeV**

## • GT

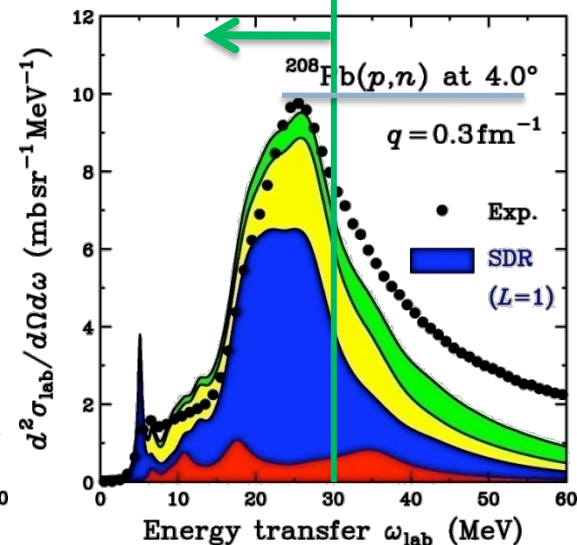
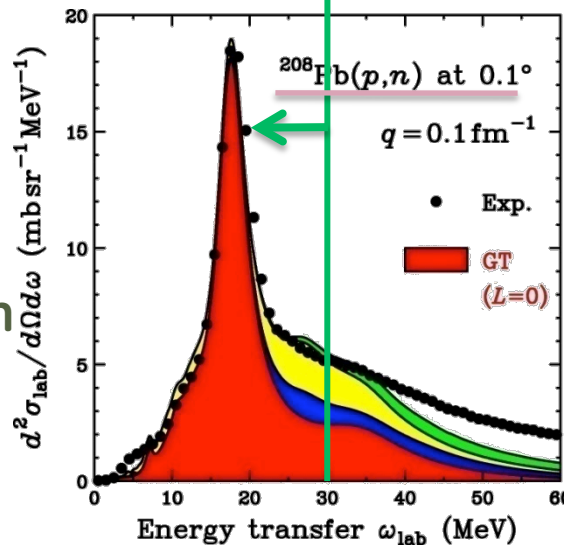
- Negligible B.G. ( $L \geq 1$ ) in GTGR
- **B(GT) can be deduced w/o MDA (Not S(GT))**

## • SD

- Small (10-20%) BG in SDR
- **B(SD) would be deduced w/o MDA with  $\sim 10\%$  error**



MDA



DWIA

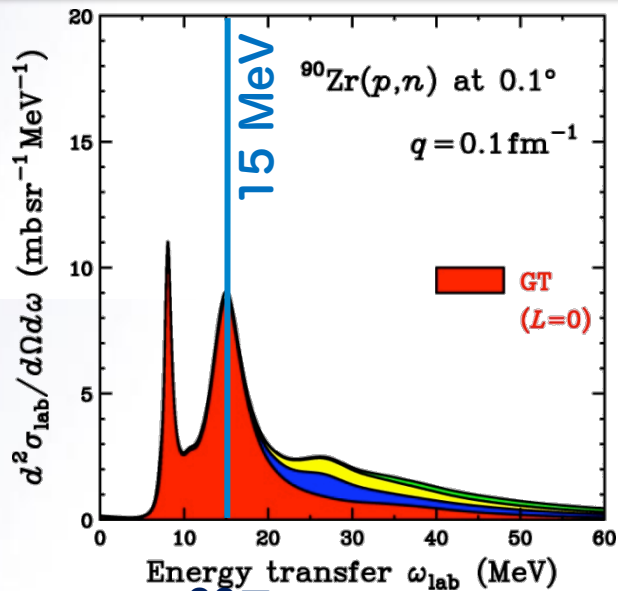


# Isotope Dependence of GT for Zr

- **Excess neutron effects on GTR**

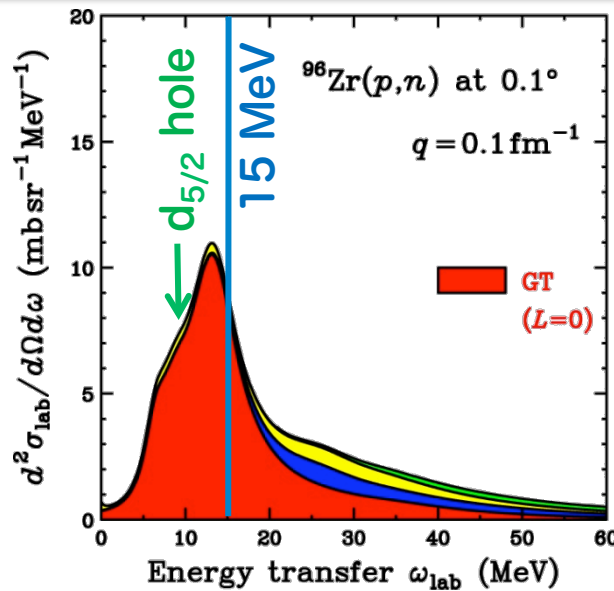
- **$^{96}\text{Zr}$  :  $d_{5/2}$  hole states fill the valley**
  - GT states form a single resonance peak
- **$^{104}\text{Zr}$  : Lower GTR peak position**
  - Due to lower proton single particle states (Larger binding energy for protons)

Information on s.p. states is important for investigating correlation effects



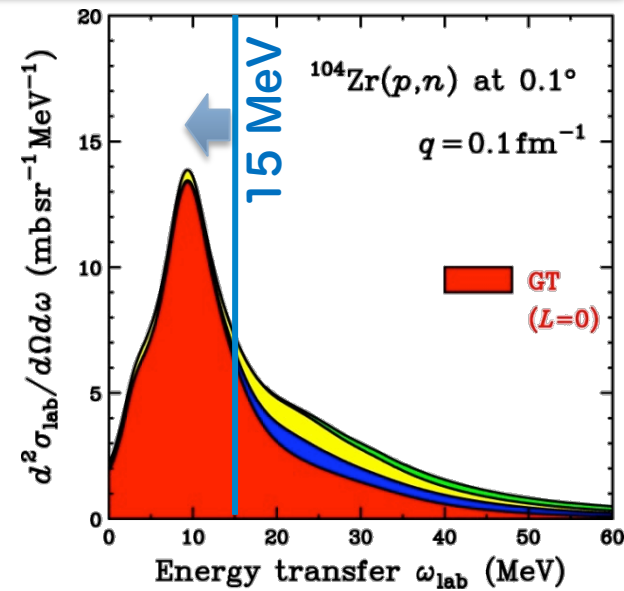
$^{90}\text{Zr}$

$g_{9/2}$  filled



$^{96}\text{Zr}$

$d_{5/2}$  filled



$^{104}\text{Zr}$

$g_{7/2}$  filled



# Isotope Dependence of SD for Zr

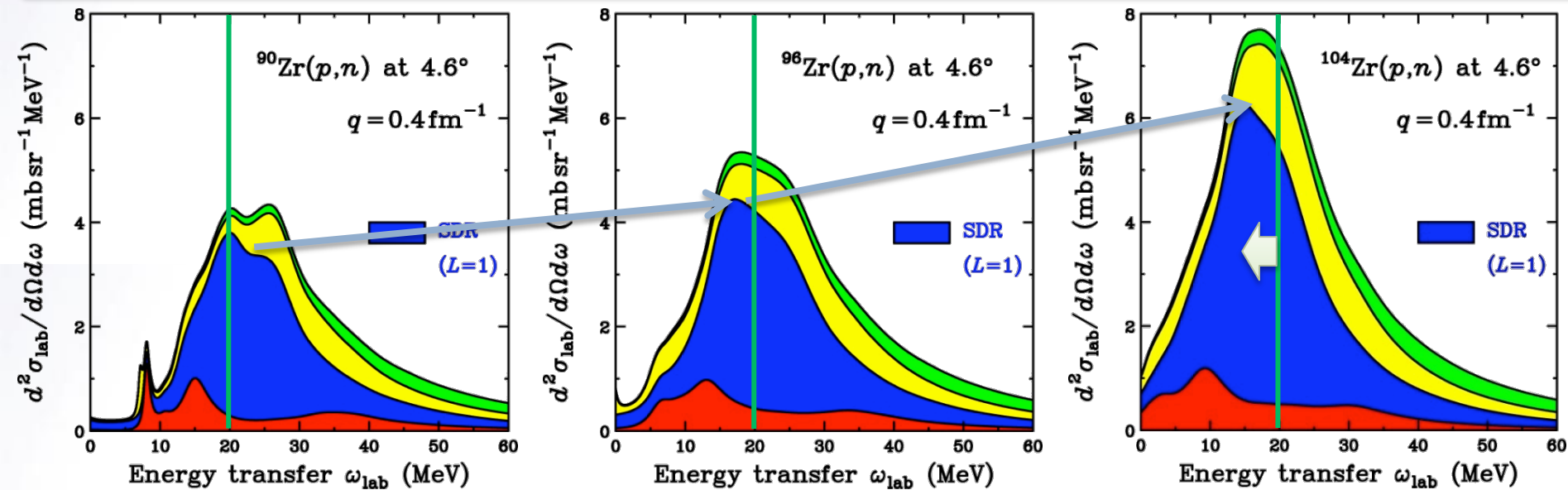
isotope dependence of SD for Zr

- **Excess neutron effects on SDR**

- SDR strengths are increased as a function of (N-Z)
- SDR peak positions are lowered due to lower proton s.p. energy

SDR strengths could be deduced by using a proportionality ansatz

- ✓ Neutron skin thickness might be deduced
- ✓ SDR distributions would give information on interaction beyond  $\pi + \rho + g'$



$^{90}\text{Zr}$   
 $g_{9/2}$  filled

$^{96}\text{Zr}$   
 $d_{5/2}$  filled

$^{104}\text{Zr}$   
 $g_{7/2}$  filled



# Neutron Skin Dependence of GT for $^{104}\text{Zr}$

- Neutron skin effects on GT/IVSM

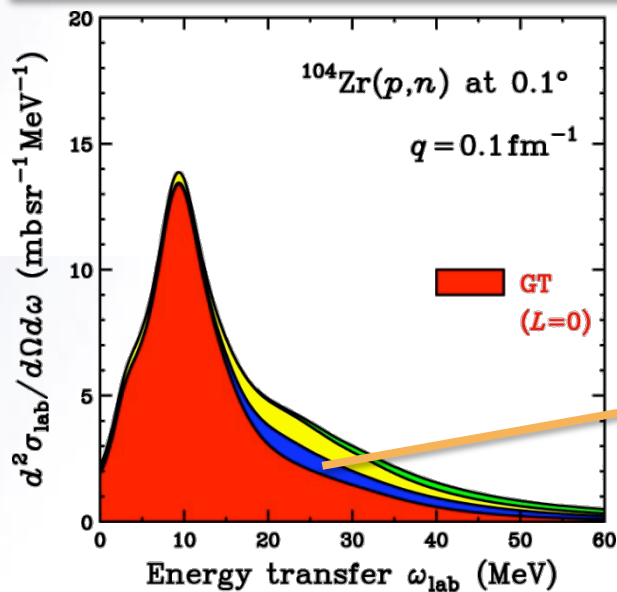
- GT : Peak height would be lowered (Interference by IVSM (?))

- IVSM : Very sensitive to neutron skin thickness

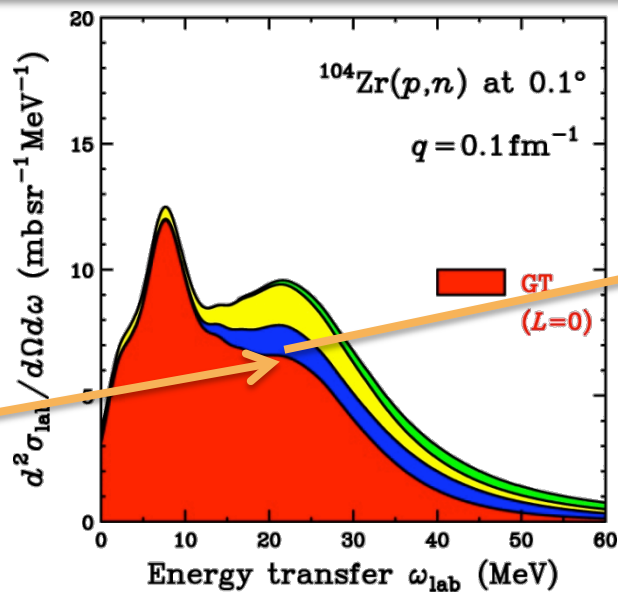
IVSM would be CLEARLY observed for RI with neutron skin

- ✓ IVSM strength is sensitive to neutron skin thickness

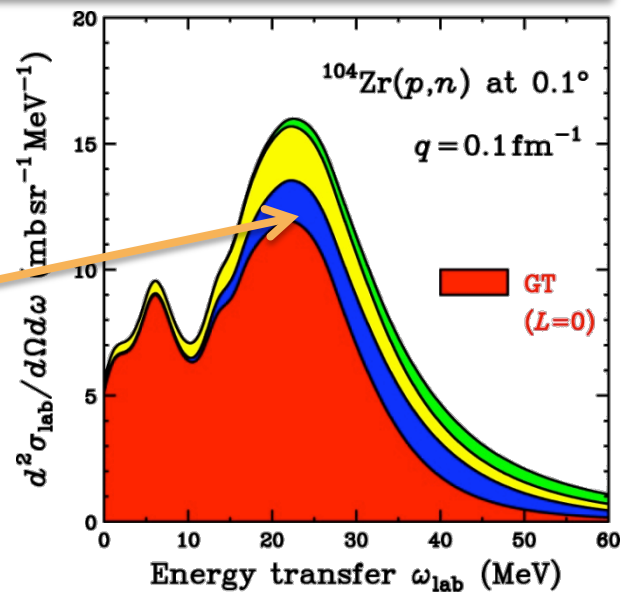
- ✓ Peak energy  $\omega \sim 20$  MeV (Very low compared with stable nuclei)



$$r_n = r_p$$



$$r_n = 1.1 \times r_p$$



$$r_n = 1.2 \times r_p$$



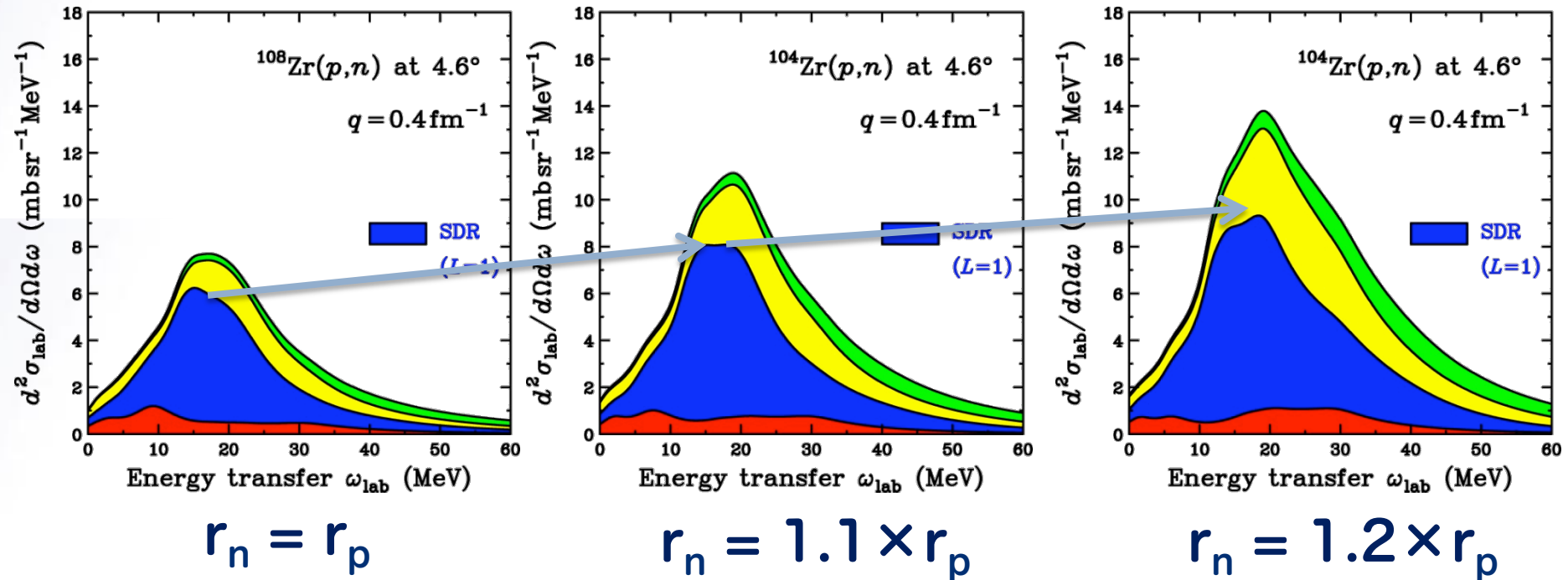
# Neutron Skin Dependence of SD for $^{104}\text{Zr}$

- Neutron skin effects on SDR

- SDR strengths are increased as a function of  $N\langle r_n^2 \rangle - Z\langle r_p^2 \rangle$
- Neutron skin effects are NOT so large

- Effects are comparable with uncertainty of proportionality ansatz

Systematic studies would be required to deduce the neutron skin effects  
 ✓ Compare with the data for normal nuclei/isotope without neutron skin



# $g'_{NN}$ Dependence of GT for $^{104}\text{Zr}$

- $g'_{NN}$  dependence for GTR

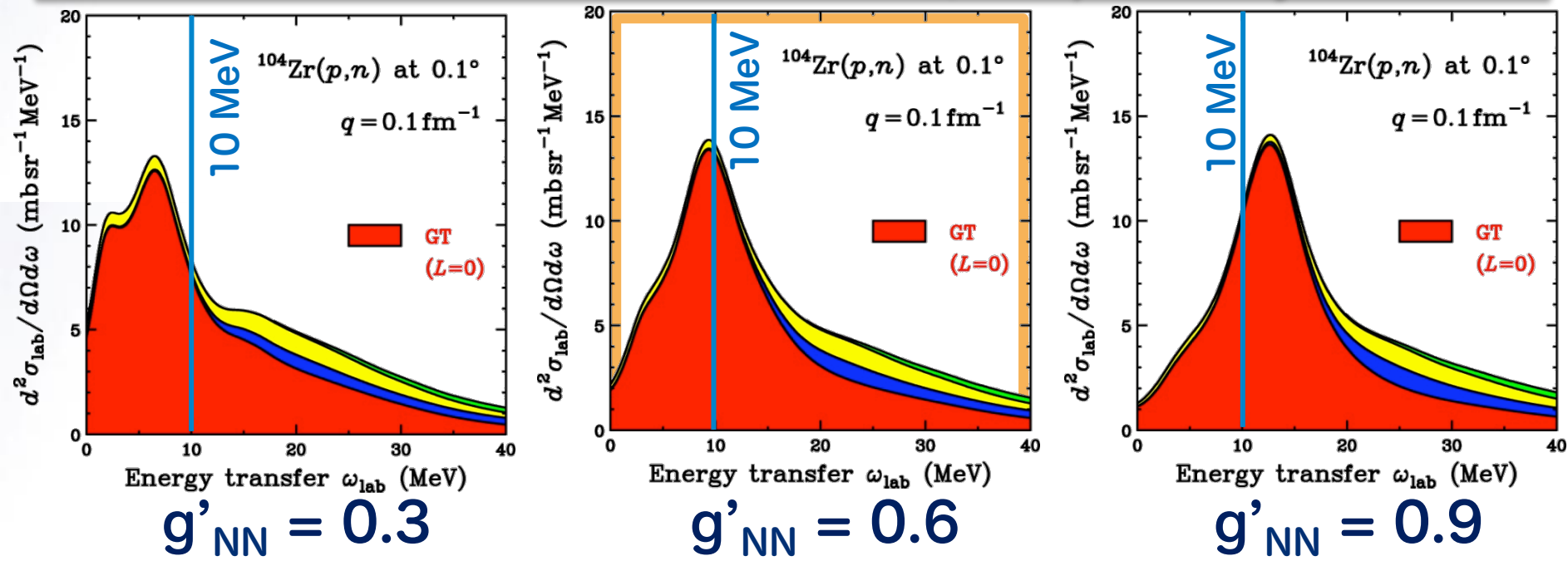
- GT strength is hardened as a function of  $g'_{NN}$
- GT strength is ALSO a function of neutron skin thickness

IVSM is sensitive to neutron skin thickness

✓ GTR peak position is NOT sensitive to neutron skin thickness

✓ Simultaneous measurement of GT and IVSM enables us to obtain

- Information on  $g'_{NN}$  (nuclear correlation), and
- Information on neutron skin thickness, independently





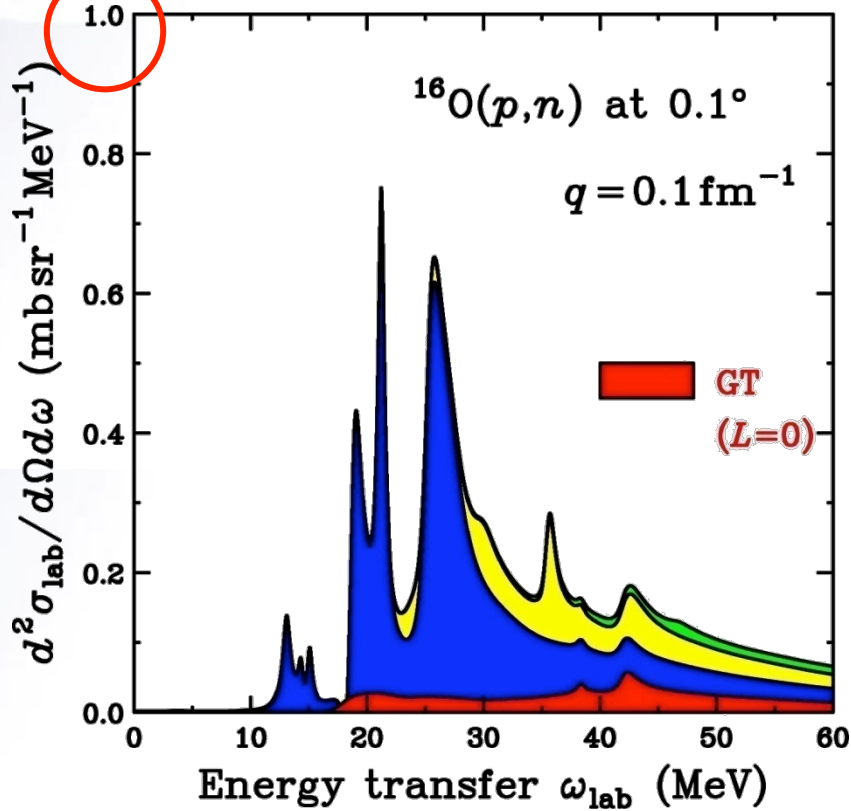
# Isotope Dependence of GT for O

## Isotope Dependence of GT for O

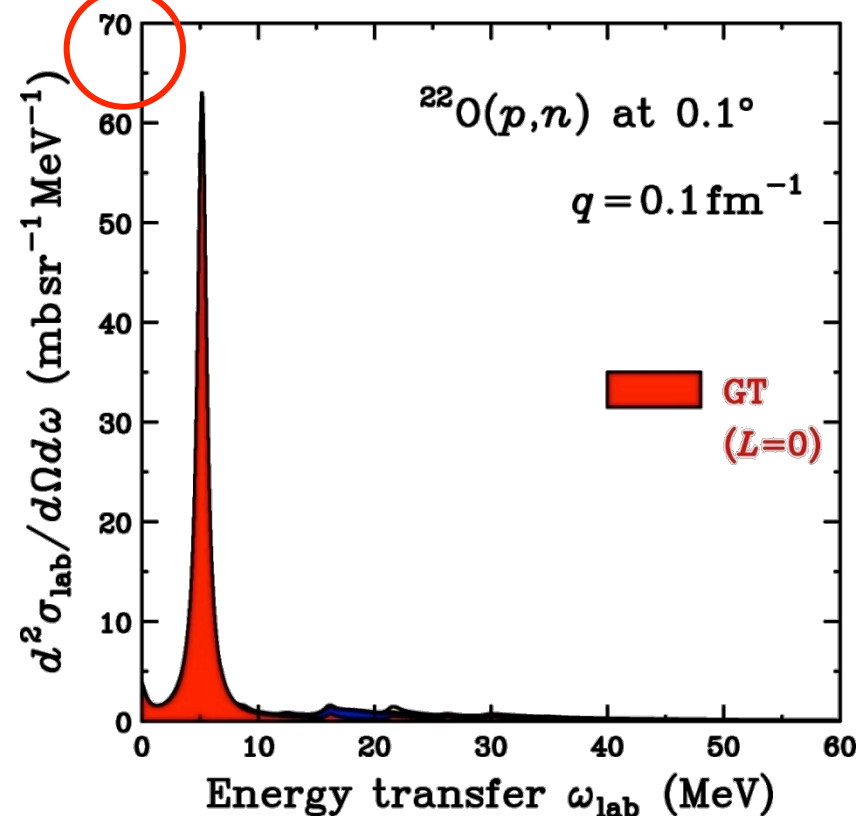
- **Excess neutron effects on GT**

- $^{16}\text{O}$  : GT strength is almost completely suppressed
- $^{22}\text{O}$  : Prominent GT peak ( $d_{5/2}$  hole) is expected

- Should be checked by comparing sophisticated calc. (S.M. etc.)



$^{16}\text{O}$  :  $p_{1/2}$  filled

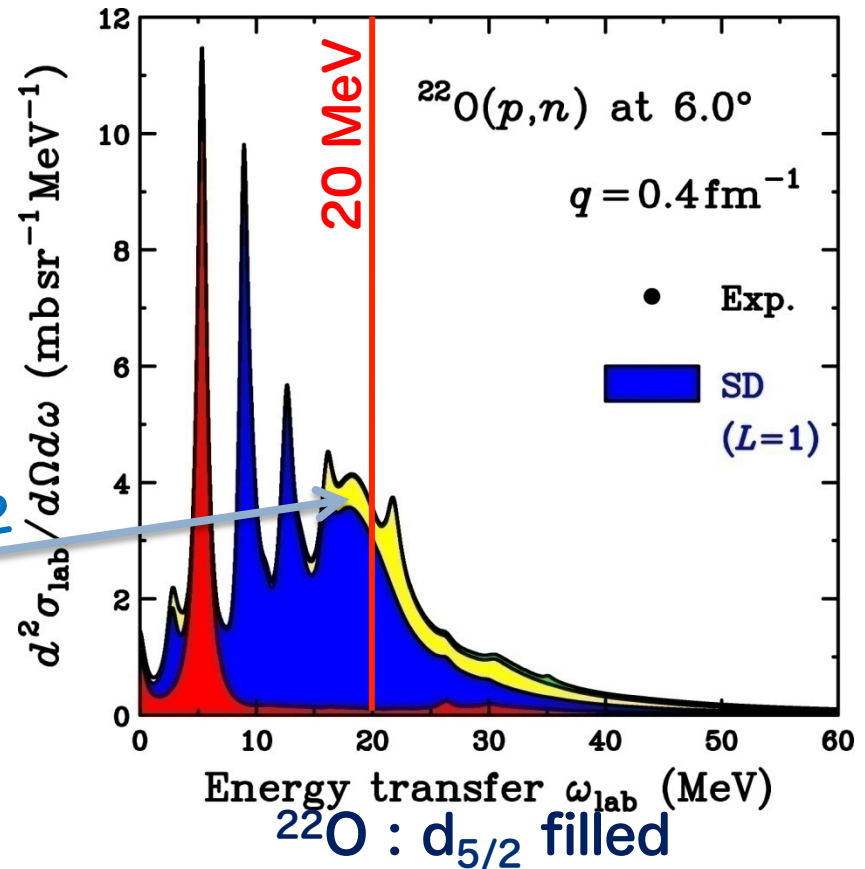
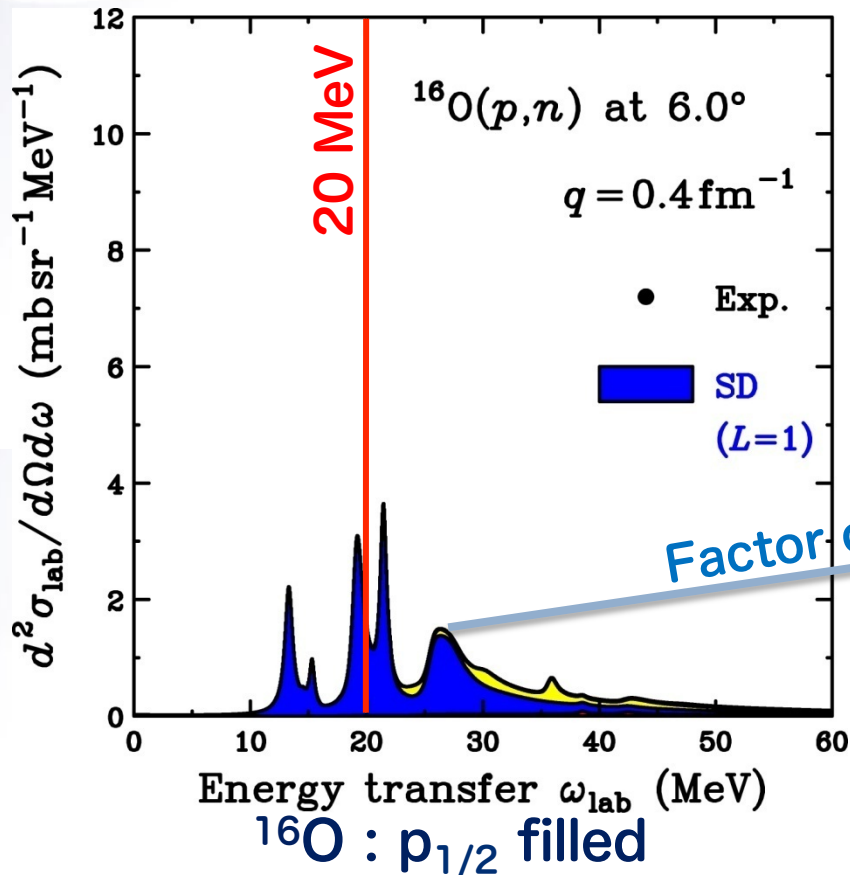


$^{22}\text{O}$  :  $d_{5/2}$  filled

# Isotope Dependence of SD for O

- **Excess neutron effects on SDR**

- SDR strengths are increased as a function of (N-Z)
  - Effects are significant compared with Zr due to larger N/Z
  - Neutron skin effects would be more clearly observed
- SDR peak positions are lowered due to lower proton s.p. energy





# Summary for Future Perspective

- **Proportionality ansatz for deducing GT/SD strengths**
  - **Reliable at  $E/A \sim 300$  MeV within  $\sim 10\%$**
- **Physical background for deducing GT/SD strengths**
  - **Less than 10-20% in p-h region ( $\leq 30\%$ )**
  - Uncertainty of strength  $\sim 10-20\%$  without MDA
    - **MDA should be performed for total strength S**
- **Prediction for Zr**
  - Excess neutron effects would be observed in GT peak
  - Excess neutron effects would be observed in SD strength
  - **IVSM would be clearly observed for nuclei with skin**
- **Prediction for O**
  - Excess neutron effects would be more clear compared with Zr

## Comment

- ✓ Elastic scattering data are important for reliable DW calculations
- ✓ Information on s.p. levels is important for correlation effects

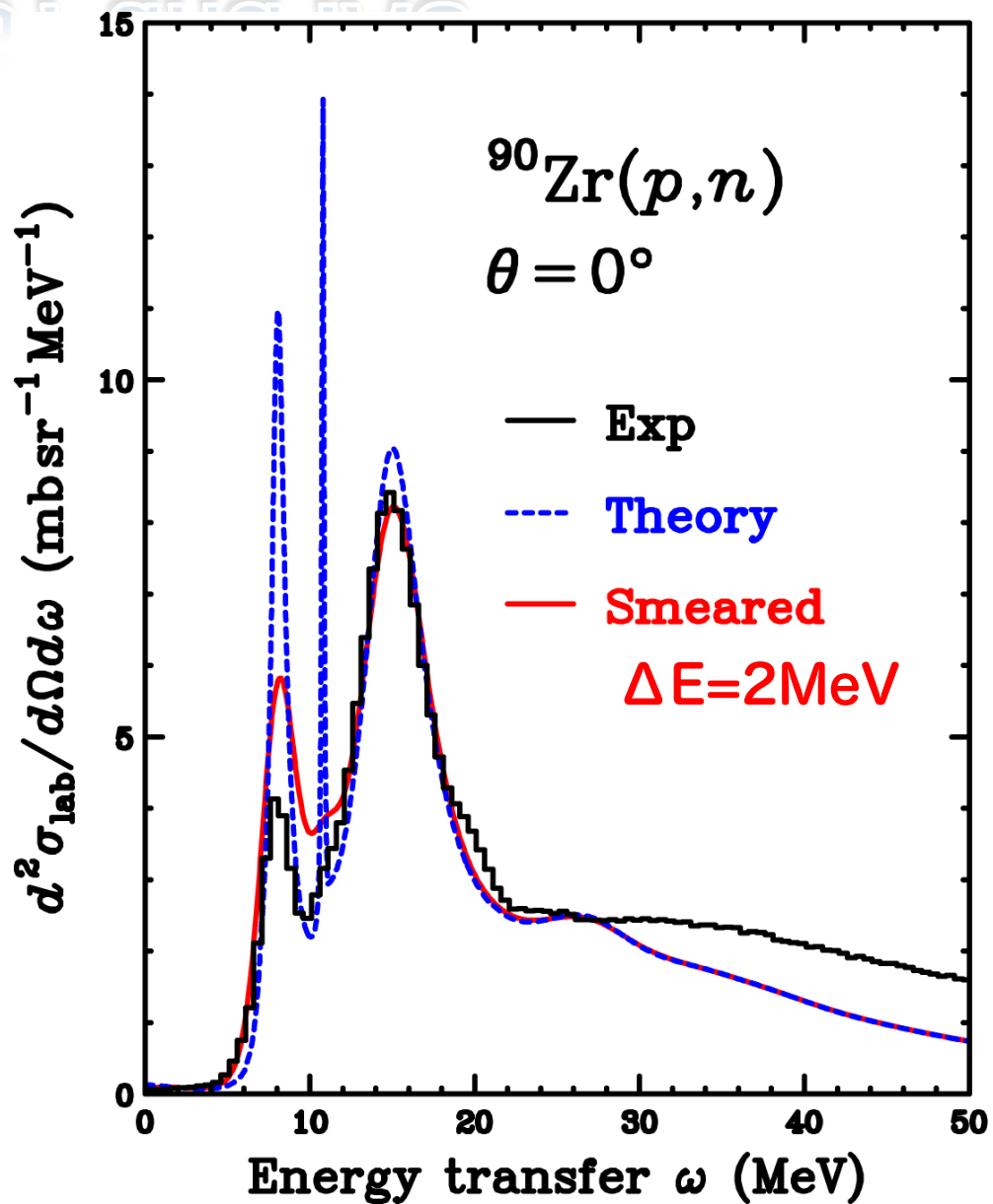


**BACKUP**

**ВУСКУБ**

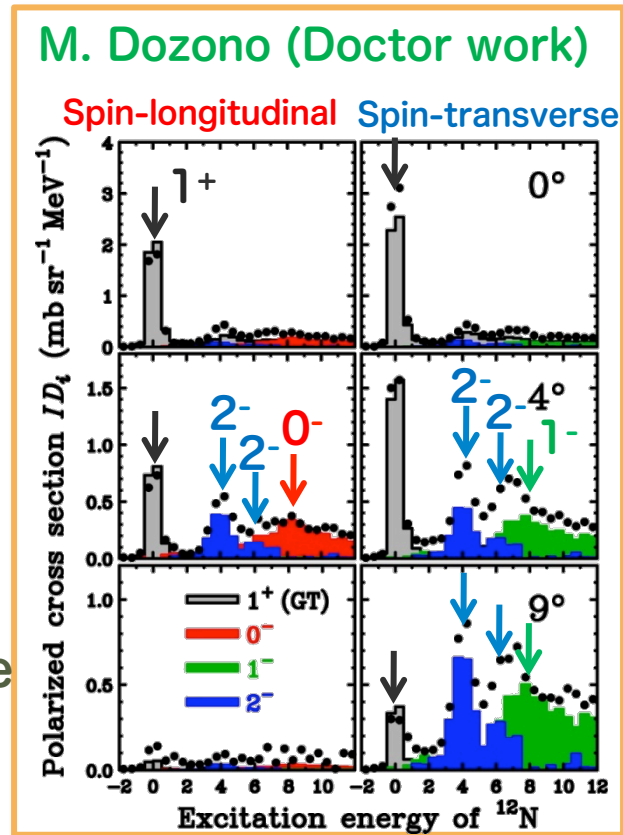


# GT and IAS



# Summary and Future Perspective

- First attempt to perform MDA with polarization data
  - Successful separation of SDR into each  $J^\pi$  component
- Information on SD strengths
  - SD strengths are NOT quenched
    - 0<sup>-</sup> strength : Fragmented
    - 1<sup>-</sup> strength : Softened  
(Inconsistent with tensor effects)
    - 2<sup>-</sup> strength : Roughly consistent
- Perspective (In Progress)
  - MDA with “complete” polarization data
    - 0<sup>-</sup>, 1<sup>-</sup>, 2<sup>-</sup> Separation becomes more reliable
    - We can check the reliability of MDA
  - RCNP-E317 :  $^{12}\text{C}(p,n)^{12}\text{N}$ 
    - Complete polarization data at 0° , 2° , 4° 6° , 9° , 12° (6 angles)



Establish method of  $J^\pi$  decomposition in continuum by polarization data

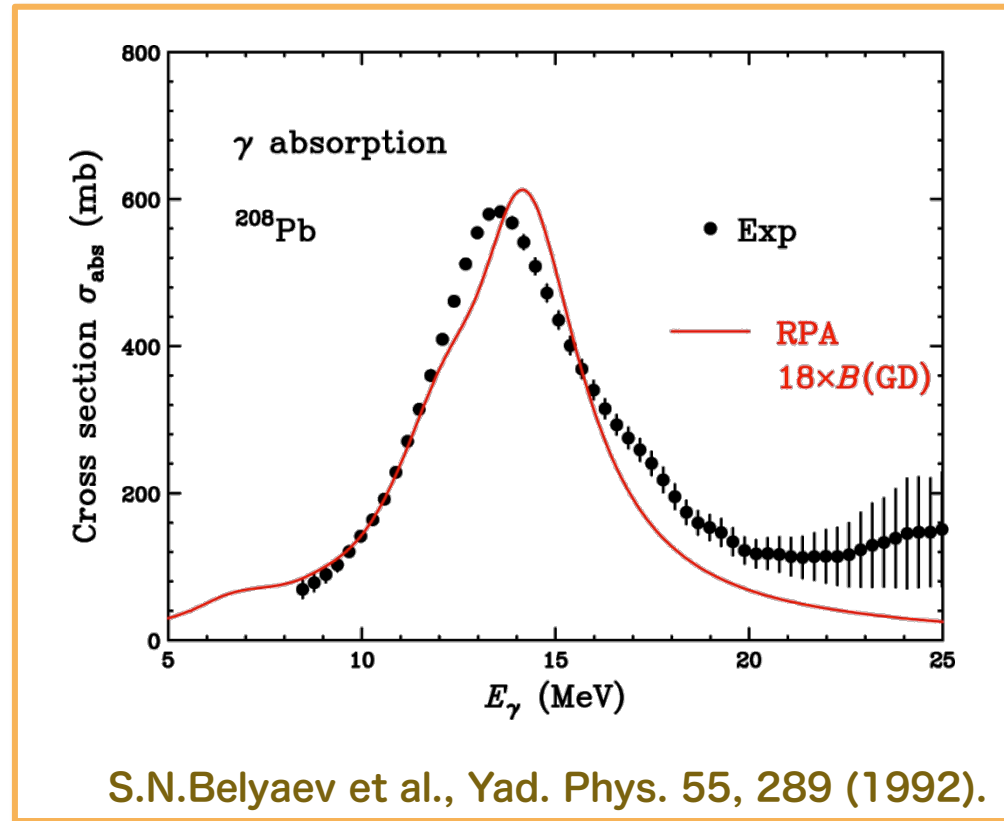
# Comment on GDR

- **Experimental GDR strength**

- $\gamma$  absorption
  - **GDR is dominant**
- $E_x \sim 14$  MeV
- $\Gamma \sim 4$  MeV

- **RPA calculation**

- **NO free parameter**
  - f is determined by IAS
- Phenomenological spreading width
- (p,n) and ( $\gamma$ ,n) difference is properly taken into account
- **Proportionality ansatz**
  - $\sigma_{\text{abs}} = 18 \times B(\text{GDR})$



GDR strength is well described by RPA

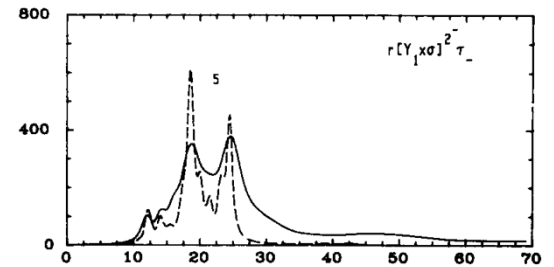
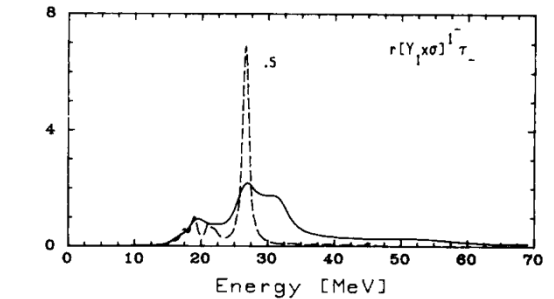
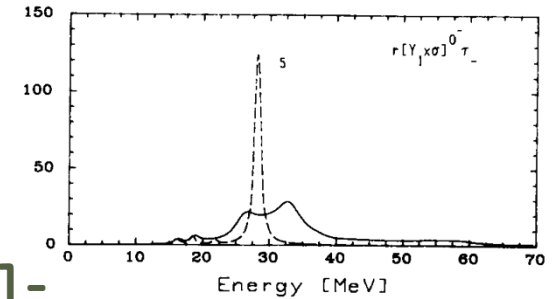
→ In MDA, GDR (non-spin) component can be fixed to RPA( $\gamma$ -abs. data)

# Previous Studies of SDR

## PREVIOUS STUDIES OF SDR

- **Theoretical predictions**

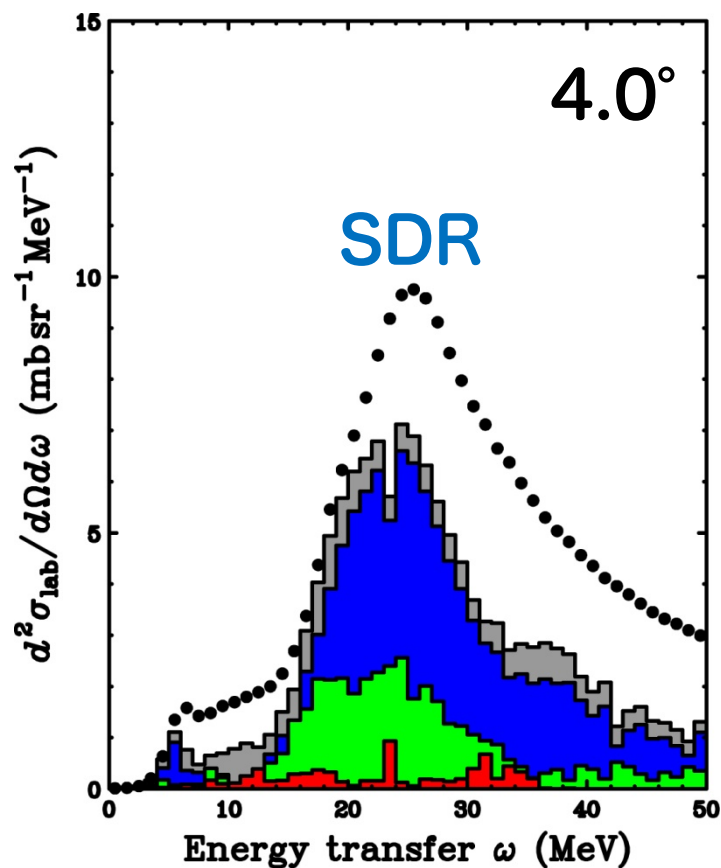
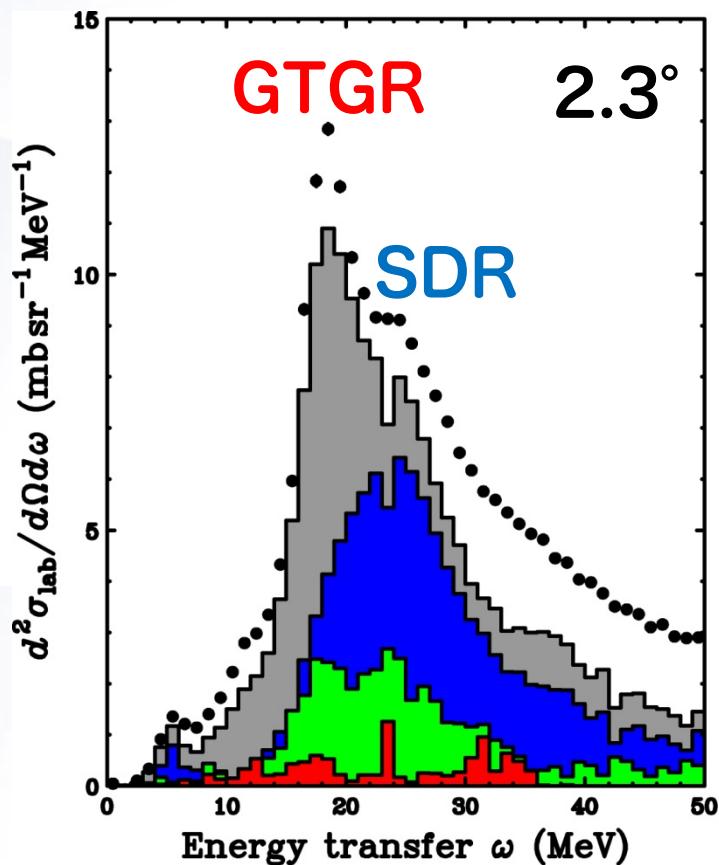
- 2p2h effects in 2<sup>nd</sup>-order RPA (SRPA)
  - Smear out RPA distributions
  - Move strengths to higher energies
- Smearing effects are relatively small in 2-
  - 2- has large config. compared with 0- and 1-
  - 2- strength is fragmented in RPA(1p1h)
  - 0- and 1- strengths → 2p2h effects





# Results of MDA for $^{208}\text{Pb}(p,n)$

MEASUREMENT OF  $^{208}\text{Pb}(p,n)$  AT  $10.5\text{ MeV}$



- 1+(GT)
- 0-(SDR)
- 1-(SDR)
- 2-(SDR)